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ADDENDUM TO "SYMMETRIZATION, SYMMETRIC STABLE PROCESSES, AND RIESZ CAPACITIES"

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In [1] we proved the following theorem.

Theorem 1. *Let $\alpha \in (0,2)$. Let K be a compact set in $\mathbb{R}^n$ with $m_n(K) = V > 0$. Let $\mathbb{B}_V$ be a ball in $\mathbb{R}^n$ with $m_n(\mathbb{B}_V) = V$. Then*

$$C_\alpha(K) \geq C_\alpha(\mathbb{B}_V).$$

Here $m_n(K)$ is the n-dimensional Lebesgue measure and C_α is the α-capacity defined by M. Riesz (see e.g. [2]).

I, as well as several other people who work on such symmetrization results in potential theory, thought that this was an open problem. T. Watanabe informed me that in his paper [3] from 1983 he proved an isoperimetric inequality (Theorem 1) for certain Lévy processes that include the symmetric stable processes. Moreover, his Theorem 2 contains an equality statement for the isoperimetric inequality. In the introduction of [3], Watanabe points out that his Theorems 1 and 2 hold for Riesz capacities, i.e. Theorem 1 above follows from Theorem 1 in [3]. The proof in [3] uses Fukushima's theory of Dirichlet forms and is completely different from the proof of Theorem 1 in [1].

References

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